

Higgsino mass matrix ansatz for MSSM

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Abstract

An ansatz, $\text{Det. } M_{\tilde{H}} = 0$, for the Higgsino mass matrix in string orbifold trinification is suggested toward the minimal supersymmetric standard model(MSSM). Small instanton solutions effective around the GUT scale can fulfil this condition. An argument that the couplings contain a moduli field is given for a dynamical realization of this Higgsino mass matrix ansatz.

[Key words: MSSM, orbifold, trinification, Higgsino mass]

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I. INTRODUCTION

The current studies on supersymmetric standard model are based on an implicit assumption that the minimal supersymmetric standard model(MSSM) may be derivable from a fundamental theory valid at a very high energy scale. We can list three important theoretical problems in such low energy supergravity models:

- The μ problem [1],
- The doublet-triplet splitting problem, and
- The problem of the MSSM construction from a fundamental theory.

The twenty year old μ problem has two well-known solutions, a dimension-4 superpotential [1] along with a Peccei-Quinn symmetry and a softly broken supergravity effect supplied with some symmetry [2]. In any case, there must be some kind of symmetry to forbid a large μ term [3]. Ever since writing a supersymmetric GUT [4], the doublet-triplet splitting problem has become one of the most fundamental problems in supersymmetric model building. In string orbifolds, there are examples that colored chiral fields beyond those in the standard model(SM) do not appear [5], which is a kind of a solution for the doublet-triplet splitting problem. However, the old *standard-like* models suffered from the unification problem in that generally the bare value of $\sin^2 \theta_W \neq \frac{3}{8}$ [6]. In addition, the number of Higgs doublets is more than one pair of H_u and H_d . This problem of more than one pair of Higgs doublets violates the gauge coupling unification. Only the MSSM spectrum, i.e. only one pair of Higgs doublets at the electroweak scale, is consistent with the gauge coupling unification with the bare value of $\sin^2 \theta_W^0 = \frac{3}{8}$.

Therefore, this problem is listed above as the problem of *MSSM construction*. Certainly this MSSM problem is a kind of the μ problem, since we must have a light Higgs doublet pair. So, the three problems listed above are all related.

In string constructions, usually there appear many Higgs doublets. In particular, the most attractive Z_3 orbifold models allow multiples of three pairs of Higgs doublets. Recently,

it has been pointed out that Z_3 orbifold models with a Wilson line(s) resulting in three families of trinification spectrum in the gauge group $SU(3)_1 \times SU(3)_2 \times SU(3)_3$,

$$\Psi_{\text{tri}} \equiv (\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1}) + (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) + (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}}) \quad (1)$$

can be the connecting theory between string scale and the MSSM scale [6]. Without a Wilson line, it was not possible to obtain the trinification spectrum [7].

To discuss the fields in terms of the SM gauge quantum numbers, let us follow the notation given in [8,9],

$$\begin{aligned} (\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1}) = \Psi_l \longrightarrow \Psi_{(\bar{M}, I, 0)} &= \Psi_{(\bar{1}, i, 0)}(H_1)_{-\frac{1}{2}} + \Psi_{(\bar{2}, i, 0)}(H_2)_{+\frac{1}{2}} + \Psi_{(\bar{3}, i, 0)}(l)_{-\frac{1}{2}} \\ &+ \Psi_{(\bar{1}, 3, 0)}(N_5)_0 + \Psi_{(\bar{2}, 3, 0)}(e^+)_{+1} + \Psi_{(\bar{3}, 3, 0)}(N_{10})_0 \end{aligned} \quad (2)$$

$$(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) = \Psi_q \longrightarrow \Psi_{(0, \bar{I}, \alpha)} = \Psi_{(0, \bar{2}, \alpha)}(q)_{+\frac{1}{6}} + \Psi_{(0, \bar{3}, \alpha)}(D)_{-\frac{1}{3}} \quad (3)$$

$$(\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}}) = \Psi_a \longrightarrow \Psi_{(M, 0, \bar{\alpha})} = \Psi_{(1, 0, \bar{\alpha})}(d^c)_{\frac{1}{3}} + \Psi_{(2, 0, \bar{\alpha})}(u^c)_{-\frac{2}{3}} + \Psi_{(3, 0, \bar{\alpha})}(\bar{D})_{+\frac{1}{3}} \quad (4)$$

where $\langle N_{10} \rangle$ and $\langle N_5 \rangle$ are needed to break $SU(3)^3$ down to the SM gauge group. We introduced the above names in view of the SM fields. The above three different representations are named carrying different *humors*: *lepton-*, *quark-*, and *antiquark-humors*. In these three sets of trinification spectrum, there are three pairs of Higgs doublets.

In Ref. [8], the trinification spectrum arises in the untwisted sector. However, that model has two phenomenological problems. One is that without $\bar{\Psi}_{\text{tri}}$ in the spectrum it is difficult to achieve a reasonable neutrino mass spectrum. Also, the D-flat direction is difficult to obtain. Therefore, it is better to have six Ψ_{tri} 's and three $\bar{\Psi}_{\text{tri}}$'s instead of just three Ψ_{tri} 's so that $\bar{\Psi}_{\text{tri}}$'s also participate in the gauge symmetry breaking. In addition, at least three Ψ_{tri} 's must come from the twisted sector so that the mass matrix is not antisymmetric [10]. If the whole sets of Ψ_{tri} and $\bar{\Psi}_{\text{tri}}$ are added in addition to the three Ψ_{tri} 's for the three light families, the GUT scale value of $\sin^2 \theta_W$ is the desired $\frac{3}{8}$. However, it may be possible to obtain a realistic $\sin^2 \theta_W$ even though we start with a much smaller GUT scale value of $\sin^2 \theta_W^0$. Sometimes, it is called ‘optical unification’ [11]. In this case, one has to introduce another parameter so that the evolution of gauge couplings change appropriately between a scale M_1

and M_{GUT} due to the assumption how the vectorlike particles are removed in this region. Certainly, this proposal is not as attractive as the $\sin^2 \theta_W^0 = \frac{3}{8}$ model. However, it can have its own virtue in that it may have smaller number of particles, sacrificing $\sin^2 \theta_W^0 = \frac{3}{8}$. In the following, we present such a model, present an ansatz for the Higgsino mass matrix, and discuss some related issues.

II. JUST THREE MORE LEPTON HUMORS

In Z_3 orbifold compactifications, one needs Wilson lines to have the trinification spectrum [6]. One interesting model has been studied before [8], but there the trinification families appear from the untwisted sector. In general, the mass matrix of the untwisted sector is antisymmetric from the H -momentum rule [12,13], and we have an unacceptable relation $m_t = m_c$ [10]. Therefore, it is better to have three families from the twisted sectors. With two Wilson lines, indeed there exist three family models from the twisted sector. The shift vector and Wilson lines are¹

$$\begin{aligned} v &= (0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{2}{3})(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0) \\ a_1 &= (\frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ 0 \ 0 \ \frac{1}{3} \ \frac{2}{3} \ 0)(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \frac{2}{3}) \\ a_3 &= (0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3})(0 \ 0 \ 0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3}). \end{aligned} \tag{5}$$

The resulting gauge group is $SU(3)^3 \times [SO(8) \times SU(3)]' \times U(1)^4$. The spectrum is shown in Table 1. Note that this model is not exactly the trinification. Let us break 4 $U(1)$'s completely by VEV's of singlets at a GUT scale.

Note that the three $SU(3)$'s do not have a permutation symmetry since in the U-sector there appear three more $(\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1})$, namely only three more lepton humors, and hence it does not constitute Ψ_{tri} of Eq. (1). The lepton humor itself is not a complete trinification, and

¹We show this just for the sake of possible realization of our ansatz, without worrying much about phenomenological obstacles such as $\sin^2 \theta_W^0 \neq \frac{3}{8}$, etc.

hence with the previous definition of the hypercharge, $Y = -\frac{1}{2}(-2I_1 + Y_1 + Y_2)$, three more lepton doublets survive down to low energy. To make the theory closer to the SM spectrum, we must make the extra lepton doublets(of the lepton humor of the untwisted sector) vectorlike after breaking the trinification group to the SM gauge group. For this purpose, we interpret that the $SU(3)_h$ from the hidden sector also contributes to the electroweak hypercharge. QCD is fixed to be the third $SU(3)_3$. Choosing the second $SU(3)$ as the weak $SU(3)$, the electroweak hypercharge is defined as,

$$Y = -\frac{1}{2}(-2I_1 + Y_1 + Y_2 + 2I_h - Y_h) \quad (6)$$

where I_i, Y_i ($i = 1, 2, h$) are the generators of $SU(3)_i$ with eigenvalues $\frac{1}{2}, -\frac{1}{2}, 0$, and $\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}$, respectively, for the fundamental representation $\mathbf{3}$. Therefore, the SM content of $(\mathbf{1}, \mathbf{\bar{3}}, \mathbf{1})(\mathbf{1}, \mathbf{\bar{3}})$ of T8 is the opposite of the lepton humor and can be considered as an antilepton humor $\bar{\Psi}_l(T8)$. However, this redefinition of the electroweak hypercharge changes the electromagnetic content of the spectrum, and hence changes the bare value of the weak mixing angle from $\frac{3}{8}$ to a value not equal to $\frac{3}{8}$, as will be shown later. Eq. (6) tolerates $\sin^2 \theta_W^0 \neq \frac{3}{8}$ and removes all the vectorlike representations at a GUT scale, instead of keeping $\sin^2 \theta_W^0 = \frac{3}{8}$ and allow three more lepton doublets at low energy. Phenomenologically, (6) seems to be better than the other choice. Of course, a model with six Ψ_{tri} 's and three $\bar{\Psi}_{\text{tri}}$'s keeps both merits, but it will have more particles than the model presented here.

The trinification spectrum in T0 has three families. The three $(\mathbf{\bar{3}}, \mathbf{3}, \mathbf{1})$ in the U-sector is $\Psi_l(U)$ and $(\mathbf{1}, \mathbf{\bar{3}}, \mathbf{1})(\mathbf{1}, \mathbf{\bar{3}})$ of T8 is a kind of complex conjugate representation of Ψ_l . It is as if we introduce $\bar{\Psi}_l(T8)$ in the twisted sector T8. Therefore, the neutrino mass matrix can be made realistic and one can choose a D-flat direction. For the Higgsino mass matrix, one should consider 9×9 (or 12×12 if we include the vectorlike lepton pairs from U and T8 also) mass matrix from $\Psi_l(U)$, $\bar{\Psi}_l(T8)$, and $\Psi_l(T0)$. Suppose that the vectorlike couplings remove $\bar{\Psi}_l(T8)$ and one Ψ_l , say that of U. Then, by giving VEV's to N_{10} 's and N_5 's, we can break $SU(3)^3$ to the SM gauge group. Also, we have to break $SU(3)_h$ completely, by giving VEV's to the $Y = 0$ and $SU(2)$ singlets in $(\mathbf{1}, \mathbf{\bar{3}}, \mathbf{1})(\mathbf{1}, \mathbf{\bar{3}})$ of T8. The vector-like representations

are expected to be superheavy.(But we want to have one pair of Higgs doublets light.) The standard model contents of $SU(3)_c \times SU(2)_W \times U(1)_Y$ nontrivial representations(using Eq. (6)) from E_8 is

$$\begin{aligned}
U &: \mathbf{2}_{+\frac{1}{2}}, \mathbf{2}_{-\frac{1}{2}}, \mathbf{2}_{-\frac{1}{2}}, \mathbf{1}_{+1} \\
T0 &: \mathbf{2}_{+\frac{1}{2}}, \mathbf{2}_{-\frac{1}{2}}, \mathbf{2}_{-\frac{1}{2}}, \mathbf{1}_{+1}, \overline{\mathbf{3}}_{+\frac{1}{3}}, \overline{\mathbf{3}}_{-\frac{2}{3}}, \overline{\mathbf{3}}_{+\frac{1}{3}}, (\mathbf{3}, \mathbf{2})_{+\frac{1}{6}}, \mathbf{3}_{-\frac{1}{3}} \\
T1 &: \mathbf{2}_{-\frac{1}{6}}, \mathbf{1}_{+\frac{1}{3}}, \mathbf{1}_{+\frac{1}{3}}, \mathbf{1}_{-\frac{2}{3}}, \mathbf{1}_{+\frac{1}{3}}, \mathbf{3}_0 \\
T2 &: \mathbf{2}_{+\frac{1}{6}}, \mathbf{1}_{-\frac{1}{3}}, \mathbf{1}_{-\frac{1}{3}}, \mathbf{1}_{+\frac{2}{3}}, \mathbf{1}_{-\frac{1}{3}}, \overline{\mathbf{3}}_0 \\
T6 &: \mathbf{2}_{-\frac{1}{6}}, \mathbf{1}_{+\frac{1}{3}}, \mathbf{1}_{+\frac{1}{3}}, \mathbf{1}_{-\frac{2}{3}}, \mathbf{1}_{+\frac{1}{3}}, \mathbf{3}_0 \\
T7 &: \mathbf{2}_{+\frac{1}{6}}, \mathbf{1}_{-\frac{1}{3}}, \mathbf{1}_{-\frac{1}{3}}, \mathbf{1}_{+\frac{2}{3}}, \mathbf{1}_{-\frac{1}{3}}, \overline{\mathbf{3}}_0
\end{aligned} \tag{7}$$

where $\mathbf{3}$, $\mathbf{2}$, and $\mathbf{1}$ are $(\mathbf{3}, \mathbf{1})$, $(\mathbf{1}, \mathbf{2})$, and $(\mathbf{1}, \mathbf{1})$, respectively, and the T5 fields are considered below. The SM contents of $SU(3)_h$ nontrivial representations are

$$\begin{aligned}
T3 &: (\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \overline{\mathbf{3}}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{3}) = m_{1/3}^1 + m_{-2/3}^2 + m_{1/3}^3 + \overline{m}_{-1/3}^1 + \overline{m}_{2/3}^2 + \overline{m}_{-1/3}^3 \\
T4 &: (\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{3}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \overline{\mathbf{3}}) = \overline{m}_{-1/3}^4 + \overline{m}_{2/3}^5 + \overline{m}_{-1/3}^6 + m_{1/3}^4 + m_{-2/3}^5 + m_{1/3}^6 \\
T5 &: (\mathbf{3}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{3}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{3}) = n_0^1 + n_{+1}^2 + n_0^3 + \bar{n}_{-1}^2 + n_0^4 + \bar{n}_{-1}^5 + n_0^6 + n_{+1}^5 + n_0^7 \\
T8 &: (\mathbf{1}, \overline{\mathbf{3}}, \mathbf{1})(\mathbf{1}, \overline{\mathbf{3}}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{3}) = \bar{l}_{+1/2}^1 + \overline{H}_{-1/2}^u + \overline{H}_{+1/2}^d + n_0^8 + e'_{-1} + n_0^9
\end{aligned} \tag{8}$$

where we denoted the electroweak hypercharges as subscripts. The superscripts are just names for particles. \overline{H} 's and \bar{l} in T8 are doublets. The singlets from T3, T4, and T5 form vectorlike representations, and we assume that they are heavy.

The chief contributors to the gauge symmetry breaking to the SM is by the VEV's of N_{10} and N_5 in $\Psi_l(U)$ and n_0^8 and n_0^9 of T8. Other N_{10} 's and N_5 's can contribute also, but we can assume that they are small.

The bare value of $\sin^2 \theta_W = \frac{g'^2}{g_2^2 + g'^2}$ can be calculated at the GUT scale in the following way. This is useful if everything is embedded in a simple group, or if all the relevant representations in consideration are obtainable by tensor products of some elementary(such as the fundamental representation in the $SU(5)$ model) representation. If $U(1)_Y$ is leaked to

other factor groups, the GUT value of $\sin^2 \theta_W$ is not the unification value as shown below. The definition of g' is $g'Y \equiv g_1 Y_1$ where Y_1 is properly normalized. The ratio of Y and Y_1 is $Y = CY_1$, or $g' = C^{-1}g_1$. Therefore, we obtain $\sin^2 \theta_W = \frac{1}{1+C^2}$. On the other hand, the electromagnetic charge can be similarly normalized, $eQ_{em} = e_U Q_U$ where Q_U is a universally normalized matrix. Thus, we have $e^2 \text{Tr}. Q_{em}^2 = e_U^2 \text{Tr}. Q_U^2$. Similarly, we have $g_2^2 \text{Tr}. (T_3^2) = g_U^2 \text{Tr}. (T_{U3})^2$. Note that $\text{Tr}. T_{U3}^2 = \frac{1}{2}$ for one doublet and $\frac{N}{2}$ for N doublets. $SU(2)_W$ triplets, quartets, etc. can be properly included. The unification implies $\text{Tr}. Q_U^2 = \text{Tr}. T_{U3}^2$. Thus, if the couplings are unified, i.e. $g_U = e_U$, we obtain $\sin^2 \theta_W = e^2/g_2^2 = \frac{\text{Tr}. (T_3)^2}{\text{Tr}. (Q_{em})^2}$. Thus, the bare value of $\sin^2 \theta_W$ can be calculated, using the quantum numbers of Eqs. (7,8), to give $\frac{2}{7}$ if all sets are contained in tensor products of some elementary representations. In fact, ours does not belong to this category, contrary to the case of trinification, and the bare value of $\sin^2 \theta_W$ is not equal to $\frac{2}{7}$ but turns out to be $\frac{1}{4}$.²

For the vectorlike $SU(2)$ doublets, we must consider 12 pairs: 3 pairs from U, three pairs from T0, three pairs from T8, and 3 lepton pairs from U and T8. But the essence can be discussed with just three pairs.

Let us consider the following 3×3 Higgsino mass matrix of the trinification spectrum,

$$M_{\tilde{H}} = \begin{pmatrix} b, & a, & a \\ a, & b, & a \\ a, & a, & b \end{pmatrix}. \quad (9)$$

This form of mass matrix is anticipated if the spontaneous symmetry breaking of $SU(3)^3$ to the SM gauge group occurs most symmetrically.

With two Wilson lines, the multiplicity 3 in fact corresponds to three different fixed points in the third two-torus where no gauge field is going around. So the three families of T0 in fact corresponds to three different fixed points among 27 fixed points of Z_3 orbifolds. In the orbifold vacua, the well-known trilinear Yukawa couplings are present for T_i, T_j, T_k ($i, j, k =$

²There was interest in $\sin^2 \theta_W = \frac{1}{4}$ before [14], but that was not from string theory.

all different) [12,13]. The trilinear Yukawa couplings for the fields from the same fixed points are different from the above couplings of three different T_i 's. The Higgsino mass matrix arises after assigning VEV's to N_{10} 's, N_5 's, n_0^8 , and n_0^9 . The form (9) is not a general one, but it can be a starting point.

III. HIGGSINO MASS MATRIX ANSATZ

Let us propose the following ansatz for the Higgsino mass matrix,

$$Ansatz : \quad \text{Determinant } M_{\tilde{H}} = 0. \quad (10)$$

Note that the eigenvalues of (9) are $b - a$, $b - a$, and $b + 2a$. There are two solutions of (10) with the mass matrix (9): one with $b = a$ and the other with $b = -2a$. The case $b = a$ has two zero eigenvalues and the case $b = -2a$ has one zero eigenvalue of $M_{\tilde{H}}$. In view of the mass hierarchy, one may be attempted to take the solution with $b = a$.

The starting mass matrix (9) is corrected by the shifts of VEV's of n_0^9 's, n_0^8 's, N_{10} 's, and N_5 's from their symmetric points. Suppose that their shift is not significant. Then, we expect the mass matrix is given by

$$M_{\tilde{H}} = \begin{pmatrix} b + \lambda_4\epsilon, & a + \lambda_1\epsilon, & a + \lambda_2\epsilon \\ a + \lambda_1\epsilon, & b + \lambda_5\epsilon, & a + \lambda_3\epsilon \\ a + \lambda_2\epsilon, & a + \lambda_3\epsilon, & b + \lambda_6\epsilon \end{pmatrix}. \quad (11)$$

where λ 's are $O(1)$ parameters and ϵ is expected to be $O(\frac{1}{10} \times a)$. With the mass matrix (11), the condition (10) gives the eigenvalues, 0, $O(\epsilon)$, and $O(a)$ for the masses of Higgsino pairs. Certainly, this is a solution for the μ problem, the doublet-triplet splitting problem and the MSSM problem if all the D of (3) and D^c of (4) are removed at a GUT scale.

Now, the key theoretical question is how one obtains the ansatz (10). This can be done dynamically by introducing a scalar field S , as the axion chooses the $\bar{\theta} = 0$ vacuum dynamically. Namely, we make the effective couplings as dynamical fields, and one of the moduli directions is assumed for this purpose. For simplicity of the discussion, suppose that

the moduli S contributes to the a (or b) couplings but not to the b (or a) couplings of Eq. (11). This moduli field settles at a value where $\text{Det. } M_{\tilde{H}} = 0$.

One obvious reason for $\text{Det. } M_{\tilde{H}} = 0$ is the determinantal instanton interaction [15]. This determinantal interaction is vanishing if the mass of any matter³ is zero. For this to happen, the $SU(3)$ couplings must be strong so that the instanton interactions are significant. Indeed, for any supersymmetric $SU(3)$ it is not asymptotically free for $N_F > 3N_c = 9$, which is possibly the case if we consider all the spectrum of the orbifold compactification, as manifested in the spectrum of Table 1, except for $SU(3)_h$. Above a GUT scale M_1 , the theory may not have the asymptotic freedom and becomes strong at some scale between the M_1 and the scale M_{GUT} which can be comparable to the string scale M_s . Therefore, the condition $\text{Det. } M_{\tilde{H}} = 0$ is not unreasonable. If we impose this condition, one pair of the Higgsinos is massless and survives down to the electroweak scale.

But, at a first glance there is no relevant $SU(3)$ group for the small instantons relevant for our scenario. It is because the model is very much chiral. To have non-chiral pieces, we identify the 9 component representation of T8 as $\bar{\Psi}_l$, through the linkage, $\langle(\mathbf{3}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{3})\rangle \neq 0$, using the field in T5. Then the diagonal subgroup $SU(3)_D$ of $SU(3)_1 \times SU(3)_h$ is unbroken and the 9 component representation of T8 transforms as $(\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})$ under $SU(3)_D \times SU(3)_W \times SU(3)_c$ and the Ψ_l of the untwisted sector transforms as $(\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1})$ under $SU(3)_D \times SU(3)_W \times SU(3)_c$. Considering the simplest instanton solution only, the instanton potential vanishes because the theory is still chiral. To introduce vectorlike representations absorbed by instantons, consider a subgroup $SU(2)_D \times SU(2)_W$. This can be an effective gauge group between the values of $\langle N_{10} \rangle$'s and $\langle N_5 \rangle$'s. The values of $\langle N_5 \rangle$'s are expected to be a factor of 10 – 100 smaller than the values of $\langle N_{10} \rangle$'s. Consider an instanton solution between

³In our current example the Higgsinos are the matter. For any instanton solution absorbing the SM fermions the determinantal interaction does not lead to a condition since the SM fermions are chiral.

these scales, transforming nontrivially under both $SU(2)_D$ and $SU(2)_W$. Then, it absorbs only those fermions transforming nontrivially under both $SU(2)_D$ and $SU(2)_W$. The matter connected to this instanton solution must be the Higgsino pairs of U, anti-Higgsino pairs of T8, and Higgsino pairs of T0. The lepton and antilepton doublets of U, T8, and T0 do not carry the $SU(2)_D$ quantum number, and they are not absorbed by this complex instanton. Then, the determinantal interaction of the remaining 9 pairs of Higgsinos is nontrivial and we can say that the Higgsino mass ansatz is supported by this instanton calculus.

IV. DOUBLET-TRIPLET SPLITTING PROBLEM

The form of the mass matrix of the trinification, without considering the moduli coupling, also applies to the color triplets D of Ψ_q and D^c of Ψ_a in T0. Let us call these colored particles as *triplets and tripletinos*, meaning color triplets and their superpartners. But in our model the quarks carrying $SU(3)_c$ quantum number either does not carry any other $SU(2)$ quantum number(D, D^c) or chiral(the SM quark doublets). So we do not have an argument for a determinantal interaction for the triplinos. This is the source of the doublet-triplet splitting in our model. For moduli couplings, the couplings to the triplinos must not be aligned to the couplings of Higgsinos. This kind of different couplings can be possible since the Higgsino masses arise from Ψ_l^3 and $\bar{\Psi}_l\Psi_l$, while the triplino masses arise from $\Psi_l\Psi_q\Psi_a$. Namely, the gauge group information is different for Higgsino and triplino couplings, and it is known that some moduli can distinguish gauge groups [16].

There exists the $d = 5$ proton decay operator obtained from $ql\bar{D}$ and qqD , which can be allowable by making D sufficiently heavy. Both of these couplings from the trinification spectrum can be made absent if we introduce a matter parity: $\Psi_{\text{tri}} \rightarrow -\Psi_{\text{tri}}$, and others(such as Ψ_l of U) = invariant.

V. LIGHT FERMION MASSES

In the trinification model, the Yukawa couplings of Higgsinos also apply to the Yukawa couplings for the quark and lepton families. However, the quark and lepton masses arise when the light Higgs boson obtains a VEV. The light Higgs boson is a massless component obtained from the above mass matrix. This component should be used for the quark and lepton masses. If we consider the mass matrix (9), the massive component is $H_{u,d}^{GUT} = \frac{1}{\sqrt{3}}(H_{u,d}^1 + H_{u,d}^2 + H_{u,d}^3)$ for $b = a$. The two massless components are orthogonal to this massive component. Small corrections leave only one component massless which will be the Higgs fields of the MSSM. After giving VEV's to these MSSM Higgs fields, one may also arrive at a flavor democratic mass matrix. For example, for $Q_{em} = \frac{2}{3}$ quarks,

$$M_u = \begin{pmatrix} \frac{m_t}{3}, & \frac{m_t}{3}, & \frac{m_t}{3} \\ \frac{m_t}{3}, & \frac{m_t}{3}, & \frac{m_t}{3} \\ \frac{m_t}{3}, & \frac{m_t}{3}, & \frac{m_t}{3} \end{pmatrix}. \quad (12)$$

If it is modified little bit by $O(\frac{1}{10} - \frac{1}{100})$ as the Higgsino mass Eq. (11) modifies the Higgsino mass Eq. (9), then there is a possibility to have a mass hierarchy for light fermion families: for example for the up-type quarks, m_t , $O(\text{a few times } \frac{1}{100}m_t)$, and 0. Here, it is merely a speculation, being taken in analogy with the Higgsino mass matrix. When supersymmetry is broken at the TeV scale, the u quark can generate a radiative mass which can be smaller than the c mass. Therefore, even if $\text{Det. } M_u = 0$ at tree level, it is not expected to have a massless up quark solution for the strong CP problem. A similar argument for mass hierarchy can be applied to down type quarks and charged leptons.

For neutrinos, the masses must be much smaller. It can happen through the seesaw mechanism if the $SU(2) \times U(1)$ singlets N_{10} 's and N_5 's obtain large masses near the GUT scale. If we did not introduce $\bar{\Psi}_l(T8)$, it is impossible to give large masses to N_{10} and N_5 , because $\Psi_l(T0)^3$ couplings leave them massless [17]. In our model, there appear $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})(\mathbf{1}, \bar{\mathbf{3}})$ in the T8 sector which allows couplings rendering N_{10} and N_5 superheavy masses. Thus, three neutrinos can obtain sub-eV masses.

VI. CONCLUSION

In conclusion, we proposed an ansatz for the Higgsino mass matrix toward realizing the MSSM at a high energy scale. This ansatz can be supported by the short distance dynamics relevant at the GUT scale such as the small instanton solutions. We presented this mass matrix ansatz in a trinification model from a Z_3 orbifold due to its simplicity in that only $SU(3)$'s appear. In principle, this ansatz can be used in other unification models. Non-universal moduli couplings to the trinification fields are suggested to choose the vacuum of our ansatz.

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TABLES

TABLE I. The massless spectrum of the orbifold (5).

sector	twist	multiplicity	fields
U		3	$(\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1})(\mathbf{1}, \mathbf{1})$
T0	V	9	$(\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{1})$
		3	$(\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1})(\mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}})(\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})(\mathbf{1}, \mathbf{1})$
T1	$V + a_1$	3	$(\mathbf{1}, \mathbf{3}, \mathbf{1})(\mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{3})(\mathbf{1}, \mathbf{1})$
T2	$V - a_1$	3	$(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})(\mathbf{1}, \mathbf{1}) + (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}})(\mathbf{1}, \mathbf{1})$
T3	$V + a_3$	9	$(\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{1})$
		3	$(\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \bar{\mathbf{3}}) + (\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{8}, \mathbf{1})$
T4	$V - a_3$	9	$(\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{1})$
		3	$(\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \bar{\mathbf{3}}) + (\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{1})(\mathbf{8}, \mathbf{1})$
T5	$V + a_1 + a_3$	3	$(\mathbf{3}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{3})$
T6	$V + a_1 - a_3$	3	$(\mathbf{1}, \mathbf{3}, \mathbf{1})(\mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{3})(\mathbf{1}, \mathbf{1})$
T7	$V - a_1 + a_3$	3	$(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})(\mathbf{1}, \mathbf{1}) + (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1})(\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}})(\mathbf{1}, \mathbf{1})$
T8	$V - a_1 - a_3$	3	$(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})(\mathbf{1}, \bar{\mathbf{3}})$